Math 2400: Calculus 3
Exam 2

October 12, 2011

Name: ______________________________________

The use of notes or textbooks is not allowed. Attempt all problems and clearly indicate answers. Solutions given with little or no justification may receive little or no credit.

By signing below I affirm that all work submitted is my own and that I have neither given nor received unauthorized assistance on this work.

Signature: ______________________________________

Indicate your section/instructor.

☐	Section 001	M. Moore	09:00 - 09:50
☐	Section 002	J. Englander	10:00 - 10:50
☐	Section 003	J. Hill	11:00 - 11:50
☐	Section 004	S. Limburg	12:00 - 12:50
☐	Section 005	C. Scherer	01:00 - 01:50
☐	Section 006	J. Hower	03:00 - 03:50

Question	Points	Score
1	10	
2	5	
3	5	
4	15	
5	10	
6	12	
7	8	
8	12	
9	15	
10	8	
Total:	100	

1. [10 points] In each of the following, let f and g be functions defined on a disc containing the point (a, b). Indicate whether each of the statements is true or false.

   (a) **T** **F** If $f_x(a, b)$ and $f_y(a, b)$ are defined, then f is differentiable at (a, b)

   (b) **T** **F** If $f_u(a, b)$ is defined for every vector u, then $f_x(a, b)$ and $f_y(a, b)$ are defined.

   (c) **T** **F** If f has an extremum at (a, b) then $\nabla f(a, b) = 0$.

   (d) **T** **F** If f is differentiable at (a, b) then f_x and f_y are continuous at (a, b).

   (e) **T** **F** If f is maximized at the point P subject to the constraint $g = c$, then there is λ such that $\nabla f(P) = \lambda \nabla g(P)$.

2. [5 points] Find the equation of the tangent plane to $f(x, y) = 2^x + y^3 - 3xy$ at the point $(0, 2)$.

   A) $0 = (\ln(2) - 6)x + 12y - z - 13$

   B) $0 = (\ln(2)2^a - 3b)(x - a) + (3b^2 - 3a)(y - b) - (z - c)$

   C) $0 = (\ln(2) - 6)x + 12y - z$

   D) $z = (\ln(2)2^x - 3y)(x - 0) + (3x^2 - 3y)(y - 2)$

   E) $0 = (\ln(2)2^x - 3y)(x - 0) + (3x^2 - 3y)(y - 2) - (z - 9)$

   F) $-(z - 9) = (\ln(2) - 6)x + 12(y - 2)$

   G) None of the above.

3. [5 points] Given $df = y^2 e^{xy^2}\, dx + 2xye^{xy^2}\, dy$, compute ∇f.

4. [15 points] Find the points on the sphere of radius 1 centered at the origin that are closest to and farthest from the point $(1, -1, 1)$.

5. [10 points] Let $f(x, y, z) = xyz$.

(a) What is the gradient of f at the point $(1, 2, 3)$?

(b) What is the directional derivative of f at $(1, 2, 3)$ in the direction of $2i + 3j + 4k$?

(c) Find a vector v such that $f_v(1, 2, 3)$ is maximized.

6. [12 points] Find all points in the xy-plane at which the direction of the greatest rate of increase of $f(x,y) = x^2 + y^2 - 2x - 4y$ is in the direction of $i + j$.

7. [8 points] Show that for any real number k the function $f(x) = e^{kx}\cos(ky)$ satisfies the partial differential equation

$$\frac{\partial^2 f}{\partial x^2} = -\frac{\partial^2 f}{\partial y^2}.$$

8. [12 points] Let

$$f(x,y) = \begin{cases} \dfrac{2xy}{(x^2+y^2)^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{otherwise.} \end{cases}$$

(a) Calculate the partial derivatives of f for $(x,y) \neq (0,0)$.

(b) Is f differentiable at all points $(x,y) \neq (0,0)$? Why or why not?

(c) Find $f_y(0,0)$.

9. [15 points] Find all critical points of $f(x, y) = x^3 + y^3 - 3x^2 - 3y + 10$ and classify each one as a local maximum, local minimum, saddle point, or none of these.

10. [8 points] Let $F(x, y, z)$ and $z = f(x, y)$ be differentiable functions such that

$$F(x, y, f(x, y)) = 0.$$

Find $\dfrac{\partial z}{\partial x}$ in terms of $\dfrac{\partial F}{\partial x}$ and $\dfrac{\partial F}{\partial z}$.