Math 2400: Calculus 3
Exam 3

November 9, 2011

Name: _____

The use of notes or textbooks is not allowed. Attempt all problems and clearly indicate answers. Solutions given with little or no justification may receive little or no credit.

By signing below I affirm that all work submitted is my own and that I have neither given nor received unauthorized assistance on this work.

Signature: _____

Indicate your section/instructor.

☐	Section 001	M. Moore	09:00 - 09:50
☐	Section 002	J. Englander	10:00 - 10:50
☐	Section 003	J. Hill	11:00 - 11:50
☐	Section 004	S. Limburg	12:00 - 12:50
☐	Section 005	C. Scherer	01:00 - 01:50
☐	Section 006	J. Hower	03:00 - 03:50

Question	Points	Score
1	10	
2	8	
3	15	
4	10	
5	8	
6	15	
7	9	
8	12	
9	13	
Total:	100	

1. [10 points] Indicate whether each of the statements is true or false.

   (a) **True** **False** If $\vec{F}(x,y)$ is a smooth vector field and $\vec{r}(t)$ is a path, then $\vec{r}(t)$ is a flow line if and only if $\vec{r}(t) = \vec{F}'(\vec{r}(t))$.

   (b) **True** **False** In the spherical coordinate system, $dV = \rho^2 \sin(\phi)\, d\phi\, d\theta\, d\rho$.

   (c) **True** **False** If $f(t)$ is a linear function, then the parametric equations $x(t) = a + b\, f(t)$ and $y(t) = c + d\, f(t)$ for $-1 \leq t \leq 1$ describe a line segment.

   (d) **True** **False** If $\vec{r}(t)$ is a path with endpoints at $t = a$ and $t = b$ and $\vec{F}(x,y,z)$ a vector field then
   $$\int_{\vec{r}} \vec{F} \cdot d\vec{r} = \int_a^b \vec{F}'(\vec{r}(t)) \cdot \vec{r}(t)\, dt.$$

   (e) **True** **False** If $\vec{F}$ is a vector field and C is a smooth oriented curve that is also closed (i.e. it has the same starting and ending points), then $\displaystyle\int_C \vec{F} \cdot d\vec{r} = 0$.

2. [8 points] Let $\vec{F}(x,y) = x\vec{i} + 2y\vec{j}$. Which of the following is a flow line of $\vec{F}$? Show work justifying your answer(s).

   A) $\vec{r}(t) = (x^2/2)\vec{i} + y^2\vec{j}$

   B) $\vec{r}(t) = (x(t)^2/2)\vec{i} + y(t)^2\vec{j}$

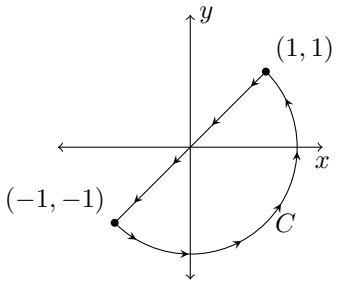
   C) $\vec{r}(t) = tx\vec{i} + 2ty\vec{j}$

   D) $\vec{r}(t) = e^t\vec{i} + 2e^{2t}\vec{j}$

   E) $\vec{r}(t) = e^x\vec{i} + 2e^{2y}\vec{j}$

   F) None of the above.

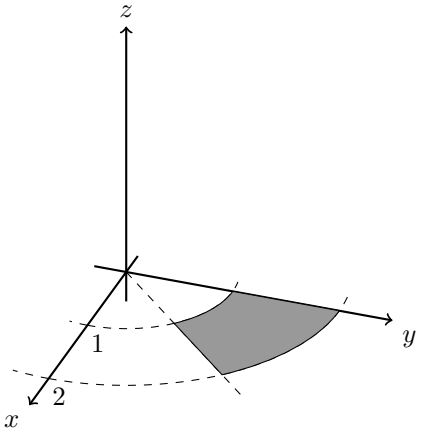
3. [15 points] Let $\vec{F}(x,y) = -y\vec{i} + x\vec{j}$ and let C be the semicircular path shown below (assume that the line segment is a diameter line of the circle). Evaluate $\int_C \vec{F} \cdot d\vec{r}$.

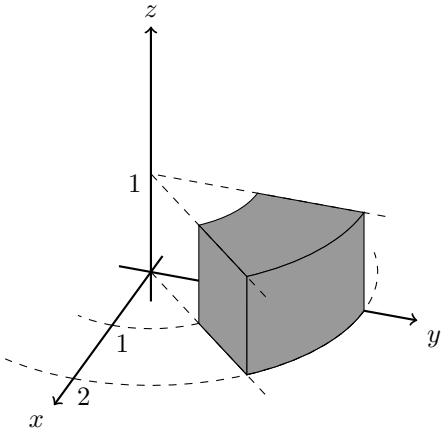


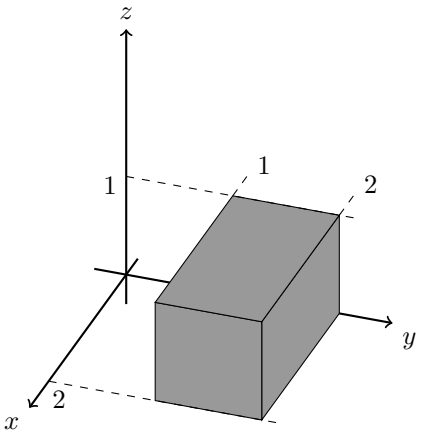
4. [10 points] Find parametric equations for the sphere given by the equation

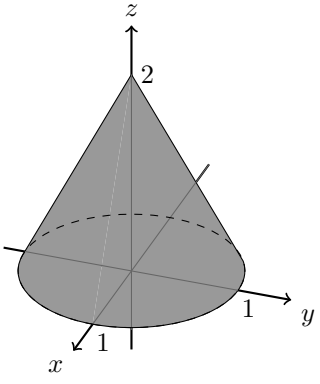
$$(x-a)^2 + (y-b)^2 + (z-c)^2 = d^2.$$

5. [8 points] For each of the following, determine whether cartesian, polar, cylindrical, or spherical coordinates should be used to determine the area/volume of the shaded region.





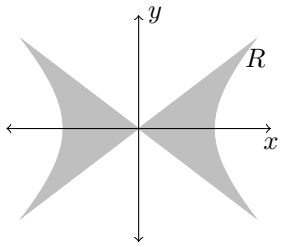




6. [15 points] Use the change of variables $x = s\cosh(t)$ and $y = s\sinh(t)$ to compute the integral

$$\int_R e^{x^2-y^2}\, dA,$$

where R is the region below given parametrically by $-1 \leq s \leq 1$ and $-1 \leq t \leq 1$. [*Hint:* $\cosh(t) = (e^t + e^{-t})/2$, $\sinh(t) = (e^t - e^{-t})/2$, *and* $\cosh(t)^2 - \sinh(t)^2 = 1$.]



7. [9 points] Suppose that a particle's position at time t is given by the parametric equation

$$\vec{r}(t) = \cos(2t)\vec{i} + \sin(2t)\vec{j} + 3t\vec{k}.$$

(a) Find the velocity vector, $\vec{v}(t)$, of the particle.

(b) Is there a time t when the particle is at rest (i.e. $\vec{v}(t)$ has magnitude 0)?

(c) How far does the particle travel over the interval $0 \leq t \leq 4\pi$?

8. [12 points] Find the volume of the solid that lies below the paraboloid $z = x^2 + y^2$ and above the region in the xy-plane bounded by the curves $y = 2x$ and $y = x^2$.

9. [13 points] Evaluate the integral $\int_0^3 \int_{y^2}^9 y\cos(x^2)\,dx\,dy$. [*Hint: sketch out the region of integration in the xy-plane, and then rewrite the integral in the form* $\int\int dy\,dx$]