MATH2400 Fall 2011 Midterm 1

Your Name:

On my honor, as a University of Colorado at Boulder student, I have neither given nor received and will not give nor receive unauthorized assistance on this exam.

Your Signature:

Fill in the box corresponding to your section:

	Section	Instructor	Time
☐	001	Moore	9:00-9:50
☐	002	Englander	10:00-10:50
☐	003	Hill	11:00-11:50
☐	004	Limburg	12:00-12:50
☐	005	Scherer	1:00-1:50
☐	006	Hower	2:00-2:50

Do Not Write Below This Line

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problem	points	score
1	6	
2	10	
3	6	
4	10	
5	8	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
Total	100	

1. True or False?

   (a) _____ If $\lim_{(x,y)\to(a,b)} f(x,y)$ exists then $f(x,y)$ is continuous at (a,b).

   (b) _____ Given two vectors $\vec{v}$ and $\vec{w}$ in 3 dimensions, $\|\vec{v}\times\vec{w}\|$ equals the area of the parallelogram formed by $\vec{v}$ and $\vec{w}$.

   (c) _____ Given two vectors $\vec{v}$ and $\vec{w}$ in 3 dimensions, $|\vec{v}\cdot\vec{w}|$ equals the area of the parallelogram formed by $\vec{v}$ and $\vec{w}$.

   (d) _____ Given two vectors $\vec{v}$ and $\vec{w}$ in n-dimensions, $\|\vec{v}+\vec{w}\| = \|\vec{v}\| + \|\vec{w}\|$

   (e) _____ Given three vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ in 3 dimensions, $\vec{u}\times(\vec{v}\cdot\vec{w})$ equals the area of the parallelopiped formed by the vectors.

   (f) _____ $f_x(x,y) = \lim_{h\to 0} \dfrac{f(x+h,y+h)-f(x,y)}{h}$

2. Given $f(x,y) = 2x^2+3y-11$, find a function $g(x,y,z)$ having $z=f(x,y)$ as a level surface at $g(x,y,z)=11$.

3. Given $h(x, y, z) = x^2 + y^2 + z^2$, describe (in words) the level surface corresponding to:

   (a) $h(x, y, z) = 1$

   (b) $h(x, y, z) = 0$

   (c) $h(x, y, z) = -1$

4. Find an equation of the plane passing through the three points:

$$(-2, 0, 0),\ (0, 1, 2),\ \text{and}\ (2, -1, 1).$$

5. Let A, B and C be three points. Let $\vec{u}$ be the displacement vector from A to B, $\vec{v}$ be the displacement vector from C to A, and $\vec{w}$ be the displacement vector from B to C. What does $\vec{u} + \vec{v} + \vec{w}$ represent? (Circle one.)

   (a) The area of the parallelogram formed by A, B, and C.

   (b) The area of the triangle formed by A, B, and C.

   (c) The displacement vector from A to C.

   (d) The sum of the areas of the parallelogram formed by A and B and the parallelogram formed by B and C.

   (e) $\vec{0}$.

6. Let $f(x, y)$ be a function such that $f(0, 0) = 1$, $f_x(0, 0) = \dfrac{3}{2}$ and $f_y(0, 0) = \dfrac{5}{4}$. Estimate the following.

   (a) $f(0.1, 0)$

   (b) $f(0, -0.1)$

7. Find the corresponding contour plot for each function. (Note that each plot is used once and only once, and the x and y-axes are in the usual orientation.)

(a) $z = e^{-x^2-y^2}$ _____

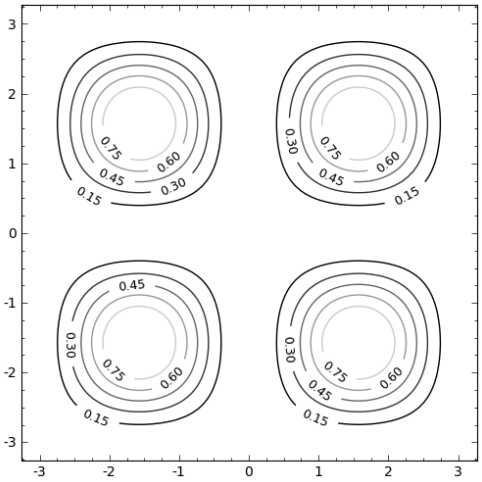
(b) $z = \sin^2(x)\sin^2(y)$ _____

(c) $z = \sin(x) + \cos(y)$ _____

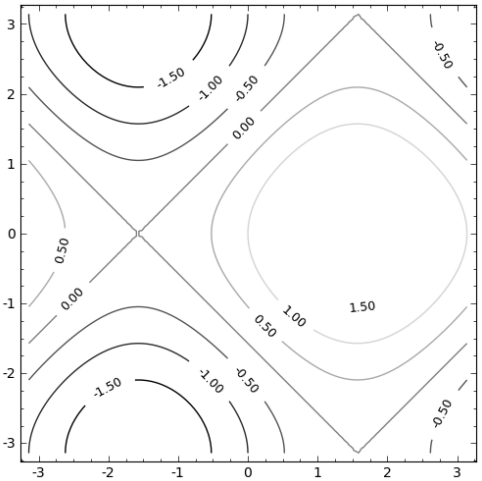
(d) $z = \sin(\sqrt{x^2+y^2})$ _____

(e) $z = xye^{-x^2-y^2}$ _____

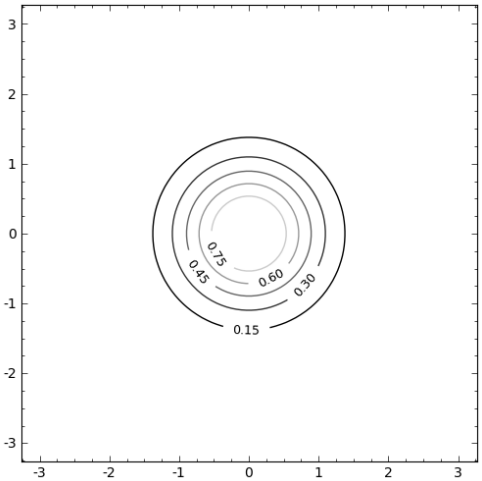
I



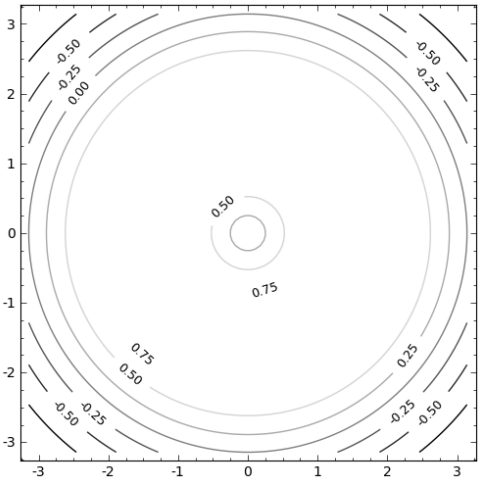
II



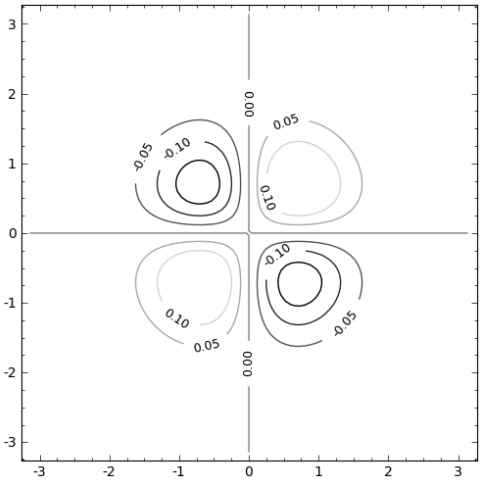
III



IV



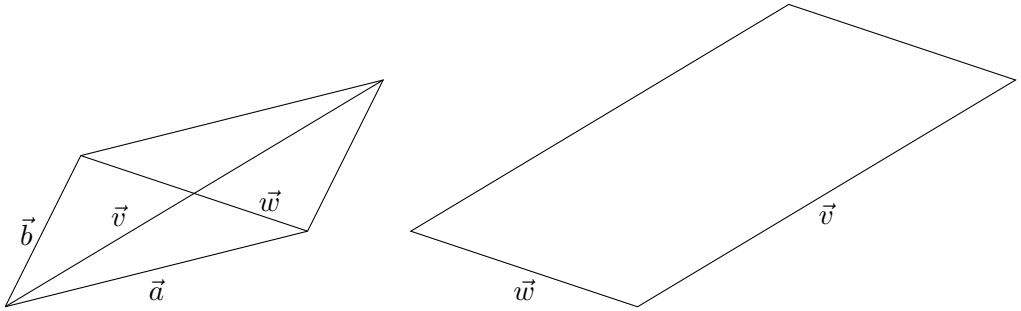
V



8. Show that $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - y^2}{x^2 + y^2}$ does not exist.

9. An old cabin, once the home of a yam farmer, in one of Colorado's ghost towns is at risk of falling down. A company is called in to examine the cabin and decides to place support wires from the top corners at the back of the cabin to the bottom corners at the front of the cabin, with the wires crossing in the middle of the single-room cabin. The two wires will be braced where they cross, but the company needs to know the angle of this crossing before constructing the brace. The front and back of the cabin are 5 meters wide, the sides are 8 meters long, and the cabin is 6 meters high. At which angle do the support wires cross?

10. Starting with the parallelogram on the left, use the diagonals $\vec{v}$ and $\vec{w}$ as edges to construct the parallelogram on the right. Note that $\vec{v} = \vec{a} + \vec{b}$ and that $\vec{w} = \vec{b} - \vec{a}$.



Using the cross product, show that the area of the parallelogram on the right is twice the area of the parallelogram on the left. (Hint: You will need to use the fact that $\vec{c} \times \vec{d} = -\vec{d} \times \vec{c}$.)

11. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the following functions.

(a) $z = e^{x+y+1}$

(b) $\arctan\left(\frac{y}{x}\right)$

(c) $z = \log_2(xy)$

(d) $z = x^y$

(e) $z = x\sin(y)$