Math 2400 Section 003 – Calculus III – Spring 2012

Chapter 12 Sections 1–4 Notes

This document is largely a collection of random notes, most of which you should have seen before in some manner. If you have questions on any of this material, do not hesitate to ask.

Equation of a Sphere

A sphere is simply a collection of points equadistant from a given point. Thus, it makes sense that its algebraic definition is given by using the Pythagorean theorem. In 2-space, we have

$$(x-a)^2+(y-b)^2=r^2 \qquad \text{or} \qquad \sqrt{(x-a)^2+(y-b)^2}=r,$$

where r is the radius of the sphere and the sphere is centered at (a,b) in the xy-plane. In 3-space, this becomes

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2.$$

Increasing/Decreasing Functions

Just as in calc 1, we have the notion of an increasing or decreasing funciton with respect to a given variable. For instance, if I consider the function $PV(n,T)=nRT$, where the function is named "PV" with input variables n and T, then we see that increases in both n and T will lead to an increase in the function's value.

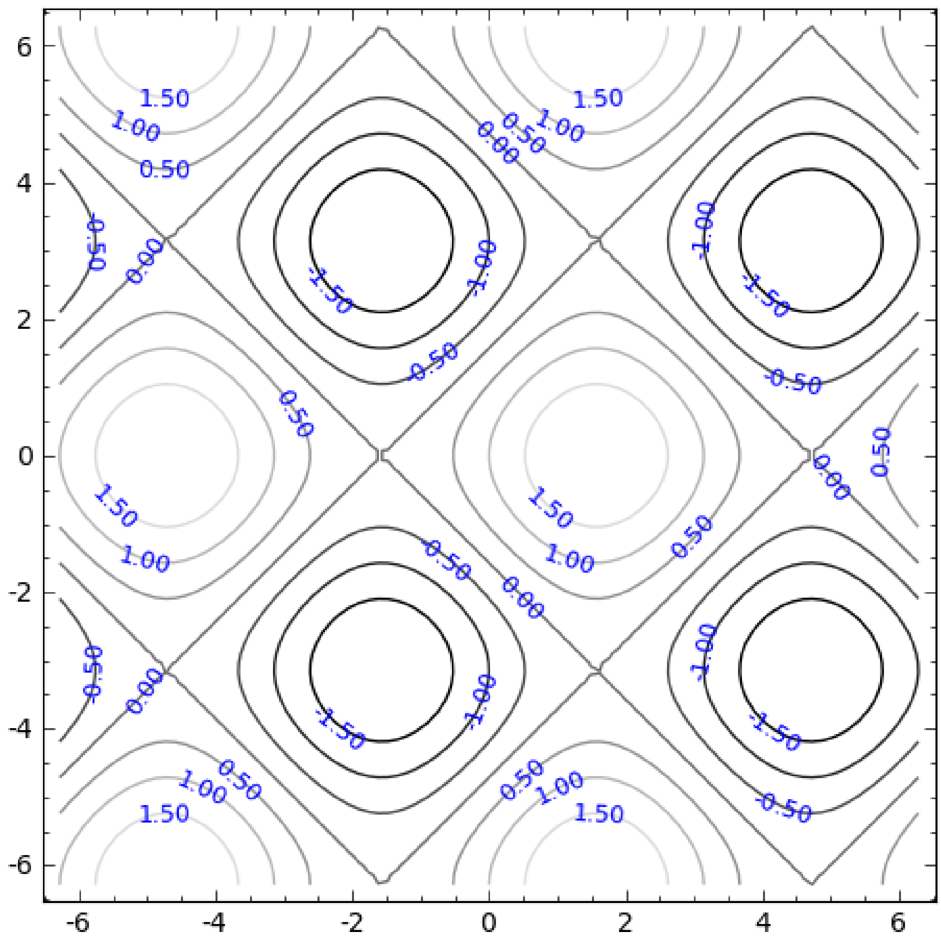
If, however, we now consider the fucntion $P(n,T,V)=nRT/V$, then this 3-variable function is increasing in n and T, but decreasing in V.

Definition of Parallel

You've heard of parallel in the context of lines. Parallel simply means "do not touch" or "do not intersect." (This definition works well in every geometric system, be it Euclidean or hyperbolic or whatever.) Thus, two planes that are parallel are planes that do not touch. In Euclidean space, being parallel is equivalent to having the same slope with respect to any coordinate axis. (This is equivalent again to the angle sum of a triangle being 180 degrees, and neither of these are true in hyperbolic geometry.) As luck would have it, this course will only use Euclidean space.

Crossing Contour Lines

Contour lines can cross. As an example, consider the function $f(x,y)=\sin x+\cos y$. The contour plot is given below. Notice that when contour lines cross, they must represent the same level set.



The Equation of a Plane

Most of our early problems in finding the equation of a plane boil down to the following sort of problem. I'll explain afterwards how this basic example can be modified.

Problem: Find the equation of a plane through the three points

$$(0,0,1),\ (0,1,2),\ \text{and } (1,1,4).$$

Solution: A plane is a linear equation, and hence has slopes (potentially in many variables), where the slopes are constant throughout the entire surface in each variable. Thus, what this really amounts to is finding the slopes, finding a point on the plane, and using the point-slope formula to find the equation we desire. (This is exactly what you do when forming the equation of a line from a point and slope. A line is a plane in one variable, afterall.)

Notice that the first two points have the same x-coordinate. As we increment y from 0 to 1 on these points, the value of z changes from 1 to 2. So, we have the ratio $\Delta z/\Delta y = 1/1 = 1$. What we are doing here is viewing z as the dependent variable and x and y as the independent variables. (The situation could be changed, but this view of z is fairly standard in 3-space.) Likewise, using the second and third points, we find $\Delta z/\Delta x = 2$. Using the point-slope form we then have

$$z - z_1 = \frac{\Delta z}{\Delta x}(x - x_1) + \frac{\Delta z}{\Delta y}(y - y_1)$$

or

$$z = \frac{\Delta z}{\Delta x}(x - x_1) + \frac{\Delta z}{\Delta y}(y - y_1) + z_1.$$

We may use any of the points given as (x_1, y_1, z_1). They will all yield the same equation for the plane. The easiest one will probably be $(0, 0, 1)$, and so we find that

$$z = 2x + 1y + 1 = 2x + y + 1.$$

You can double-check this by verifying that the points in question do indeed satisfy this equation.

The only way to make this more challenging is to ask the question differently. You could, for instance, consider a line and a point not on that line in 3-space. These would also give you a plane. In order to form that plane's equation, you'd need to come up with two points on the line, and now you're back to the same situation as in this problem. Most of our early plane equation problems take this form in some way, although it may not be obvious at first how.