Math 2400 Midterm Review 2

1. Consider the surface S determined by the equation $2x^2 + 3y^2 + z^2 = 20$.

   (a) Verify that the point $P = (2, 1, 3)$ is a point on S and find the equation of the tangent plane to S at this point.

   (b) The above equation defines z implicitly as a function of x and y, $z = f(x, y)$. Find the local linear approximation for $f(x, y)$ at $(2, 1)$.

   (c) Approximate the value of z corresponding to $x = 1.97$ and $y = 1.12$.

2. Let $w = f(\rho)$, where $\rho = \sqrt{x^2 + y^2 + z^2}$. Show that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 + \left(\frac{\partial f}{\partial z}\right)^2 = \left(\frac{\partial f}{\partial \rho)}\right)^2.$$

3. An object's specific gravity S can be found using the formula

$$S = \frac{A}{A - W}$$

   where A is the number of pounds the object weighs in air and W is the number of pounds the object weighs in water. The weight in air A is measured to be 10 ± 0.1 lbs, and the weight in water W is measured to be 8 ± 0.08 lbs. Use differentials to estimate the maximum possible error in calculating specific gravity.

4. Consider the function

$$f(x, y) = \begin{cases} \dfrac{x^3 y^2}{x^4 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

   (a) Is $f(x, y)$ continuous everywhere? If not, where is it not continuous.

(b) What are $f_x(0,0)$ and $f_y(0,0)$?

(c) Are $f_x(x,y)$ and $f_y(x,y)$ continuous?

(d) Is $f(x,y)$ differentiable everywhere? If not, where is it not differentiable.

5. An airline limits the size of luggage that a passenger can carry by requiring that the sum of the length, width, and height be at most 135 cm. Find the largest volume of luggage that a passenger is allowed to carry.

6. (a) Find a vector normal to the paraboloid $z = x^2 + y^2$ at the point $(1, 2, 10)$.

   (b) Find the tangent plane of the paraboloid $z = x^2 + y^2$ at the point $(1, 2, 10)$.

   (c) Find the minimum distance from the paraboloid $z = x^2 + y^2$ to the point $(1, 2, 10)$.

   (d) Find the vector from the point on the paraboloid you found in part (c) to $(1, 2, 10)$ What is the dot product of this vector with the vector you found in part (a)?

7. (a) What is the second order Taylor polynomial $p_2(x)$ for $f(x) = e^x$ at $x = 0$?

   (b) What is the second order Taylor polynomial $P_2(x, y)$ for $f(x, y) = e^{2x+3y}$ at $(0, 0)$?

   (c) With your answer $p_2(x)$ from part (a), what is $q(x, y) = p_2(2x)p_2(3y)$? How does $q(x, y)$ compare to your answer in part (b)?