

## Calculus 3 – Spring 2012

### Written Homework #5

Due 2/24/2012

**Problem 1)** A square metal plate with side length 2 centered at  $(0,0)$  in the  $xy$ -plane has been heated and has a temperature

$$T(x,y) = 100 - 4x^2 - 2y^2$$

at the point  $(x,y)$ . A bug is standing at the point  $(-1/3, 5/8)$  and decides its feet are cold so it starts moving in the south-east direction at a rate of 3 units per hour. Calculate the rate of change in temperature that the bug feels when it begins moving.

**Problem 2)** Let  $f(x,y,z) = x^2 + y^2 + z^2$  Find all points on the level surface  $f(x,y,z) = 1$  so that the tangent plane to the surface at the point is parallel to both the vector  $\vec{i} - \vec{j} + \vec{k}$  and the vector  $\vec{k}$ .

**Problem 3)** For any real number  $t$  define the vector  $s(\vec{t}) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$  and for a fixed value  $t_0$  define the vector  $s'(\vec{t}_0) = x'(t_0)\vec{i} + y'(t_0)\vec{j} + z'(t_0)\vec{k}$ . Suppose  $s(\vec{t}_0) = (x_0, y_0, z_0)$ . Let  $F(x,y,z)$  be any differentiable function. Prove that the directional derivative  $F_{s'(\vec{t}_0)}(x_0, y_0, z_0)$  of  $F$  at  $(x_0, y_0, z_0)$  in the direction of  $s'(\vec{t}_0)$  is the same as the derivative  $\frac{d}{dt} (F(x(t), y(t), z(t)))|_{t=t_0}$ . i.e. prove

$$F_{s'(\vec{t}_0)}(x_0, y_0, z_0) = \frac{d}{dt} (F(x(t), y(t), z(t)))|_{t=t_0}.$$

**Problem 4)** Let  $f(x,y,z) = x \cos(y) \sin(z)$ ,  $g(x,y,z) = x \sin(y) \sin(z)$ , and  $h(x,y,z) = x \cos(z)$  for  $x > 0$ ,  $0 \leq y \leq 2\pi$ , and  $0 \leq z \leq \pi$ . Calculate

$$|(\nabla f \times \nabla g) \cdot \nabla h|.$$

**Problem 5)** For any positive integer  $p$  a function  $f(x,y,z)$  is called homogeneous of order  $p$  if for any real number  $t$  the following equation holds:

$$f(tx, ty, tz) = t^p f(x, y, z).$$

Prove that if the function  $f$  is homogeneous of order  $p$  then

$$xf_x(x,y,z) + yf_y(x,y,z) + zf_z(x,y,z) = pf(x,y,z).$$