Calculus 3 - Spring 2012
Written Homework #7
Due 3/16/2012

1. Suppose $z = f(x, y)$ is a continuous function with values given in the table:

$x \backslash y$	0	2	4
-1	1	11	6
0	12	5	4
1	11	9	2
2	4	6	13

From the table, find reasonable lower and upper approximations of the volume under f on the rectangle

$$R = [-1, 2] \times [0, 4] = \{(x, y) | -1 \leq x \leq 2,\ 0 \leq y \leq 4\}.$$

Note: In this case, your estimates are reasonable if your upper and lower estimates differ by no more than 100.

2. Evaluate the following integrals. Make sure to show all your work, and give thorough explanations if necessary.

   (a) $\displaystyle\int_0^\pi \int_0^5 21x^5 \cos(x^3 y) \cos^6(\pi x^3)\, dx\, dy$

   (b) $\displaystyle\int_0^1 \int_0^{x^3} e^x \sin y\, dy\, dx + \int_0^1 \int_{\sqrt{x}}^1 e^x \sin y\, dy\, dx + \int_0^1 \int_{y^2}^{\sqrt[3]{y}} e^x \sin y\, dx\, dy$

   (c) $\displaystyle\iint_R \cos x \sin y\, dA$ where R is the region inside the circle $(x-6)^2 + y^2 = 25$ and outside the square with corners $(3, 0)$, $(5, 2)$, $(7, 0)$, $(5, -2)$.

3. Prove the following using integration:

   (a) For $a, b > 0$, the triangle with vertices $(0, 0), (a, 0), (0, b)$ has area $\dfrac{ab}{2!}$.

   (b) For $a, b, c > 0$, the tetrahedron with vertices $(0, 0, 0), (a, 0, 0), (0, b, 0), (0, 0, c)$ has volume $\dfrac{abc}{3!}$.

4. Find the solid G that maximizes the integral

$$\iiint_G 144 - 4x^2 - 9y^2 - 16z^2\, dV.$$

Explain carefully why the integral is maximized on G.

5. Rewrite $\displaystyle\int_0^1 \int_0^{\sqrt{x}} \int_{1-y^3}^1 f(x, y, z)\, dz\, dy\, dx$ as an equivalent iterated integral in the five other orders.