HW 12

Due Friday, April 27

1. Let $\vec{\boldsymbol{F}} = \left\langle 4x - 2xy\sin(x^2),\, \cos(x^2) + 5 \right\rangle$ and C be the part of the parabola $y = x^2 + 5x + 10$ from $x = 0$ to $x = -3$ followed by the arc of the semicircle $x = -\sqrt{25 - y^2}$ from $y = 4$ to $y = -3$, followed by the line segment from $(-4, -3)$ to $(5, 0)$. Compute $\displaystyle\int_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$.

2. Let $\vec{\boldsymbol{F}} = \langle g(x,y), h(x,y) \rangle = \left\langle \dfrac{-y}{x^2 + y^2}, \dfrac{x}{x^2 + y^2} \right\rangle$.

   (a) Compute $\dfrac{\partial g}{\partial y}$ and $\dfrac{\partial h}{\partial x}$. Is $\vec{\boldsymbol{F}}$ a gradient field? If so, find a function $f(x, y)$ such that $\vec{\boldsymbol{F}} = \vec{\nabla} f$.

   (b) Compute the line integral $\displaystyle\int_{C_R} \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$, where C_R is the circle of radius R, centered at the origin, oriented counterclockwise. Is $\vec{\boldsymbol{F}}$ a conservative vector field?

   (c) Explain why your answers in (a) and (b) are not a contradiction.

3. Let $f(x, y)$ be a differentiable function with continuous first order partial derivatives. Show that $\displaystyle\int_C \vec{\nabla} f \cdot d\vec{\boldsymbol{r}} = 0$ if and only if the endpoints of C lie on the same contour of f.

4. Use the vector field $\vec{\boldsymbol{F}} = \dfrac{1}{2}\langle -y, x \rangle$ and Green's Theorem to find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

5. Let $\vec{\boldsymbol{F}} = \left\langle e^{x^2} + 4x^2y,\ 144x + \sin y - 9xy^2 \right\rangle$. Find the counterclockwise oriented, simple closed curve, C, with maximal circulation in $\vec{\boldsymbol{F}}$. That is, the curve C where $\displaystyle\oint_C \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$ is maximized.