Math 2400 Midterm Review 3

1. Compute the integral $\iint_R xy\, dA$ over the region R bounded by the curves $x = 1$, $x = -1$, $x = y^2$, $y = -1$, and $y = 1 + x^2$.

2. A swimming pool is circular with a $40ft$ diameter. The depth is constant along east-west lines and increases linearly from $2ft$ at the south end to $7ft$ at the north end. Find the volume of water the pool can hold.

3. Find the volume of the solid determined by the inequalities $y \leq z^2 + x^2$ and $x^2 + y^2 + z^2 \leq 3$.

4. Evaluate the integral $\iint_R x^2 e^{\frac{x^2}{y^2}}\, dA$, where R is the region bounded by $y = \frac{1}{x}$, $y = \frac{2}{x}$, $y = x$, and $y = 2x$. *Hint:* Try a change of variables.

5. Let $\vec{F}(x, y) = \langle -x^2, 2xy \rangle$. Find an equation for the curve that goes through the point $(1, 2)$ and is perpendicular to $\vec{F}$ at every point. *Hint:* $dy/dx = y'(t)/x'(t)$.

6. Show that the flow lines of $\vec{F}(x, y) = \langle x - 2y, 6y - 3x \rangle$ are lines. Assuming a particle is at $(4, 9)$ when $t = 0$, find the acceleration vector of the particle at $t = 1$.

7. Parameterize the following surfaces:
   (a) The portion of the surface $z = x + 3$ inside the cylinder $x^2 + y^2 = 7$.
   (b) The portion of the sphere of radius 1 centered at the origin that is inside the sphere of radius 1 centered at $(0, 0, 1)$.

8. A $80kg$ man carries a $10kg$ can of paint up a helical staircase that encircles a silo with a radius of $20m$. If the silo is $90m$ high and the man makes exactly three complete revolutions in 6 minutes, how much work is done by the man against gravity in climbing to the top. *Hint:* The force of gravity on an object is $\vec{F} = -mg\vec{k}$ where m is the mass of the object and $g \approx 9.81m/s^2$ is the acceleration due to gravity.

9. Let C_1 be the line segment from (2,3) to (5,9), And C_2 be the portion of $x^2 + y^2 = 4$ going from $(1, \sqrt{3})$ to $(-2, 0)$ Compute the following:
   (a) $\int_{C_1} a\, dx + b\, dy$
   (b) $\int_{C_1} \sin\left(\frac{\pi}{3} y\right)\, dx + \cos\left(\frac{2\pi}{3} x\right)\, dy$
   (c) $\int_{C_2} x\, dx + y\, dy$
   (d) $\int_{C_2} y\, dx - x\, dy$