WeBWorK assignment number ExamReview is due : 05/09/2012 at 05:00am MDT.

The

(* replace with url for the course home page *)

for the course contains the syllabus, grading policy and other information.

This file is /conf/snippets/setHeader.pg you can use it as a model for creating files which introduce each problem set.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don't spend a lot of time guessing – it's not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $sin(3*pi/2)$instead of -1, $e \wedge (ln(2))$ instead of 2, $(2+tan(3))*(4-sin(5)) \wedge 6-7/8$ instead of 27620.3413, etc. Here's the **list of the functions** which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

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1. **(0 pts) Library/CU/Michigan/Chap12Sec1/Q31v2.pg**

Find the equations of planes that just touch the sphere $(x-1)^2+(y-1)^2+(z-5)^2=25$ and are parallel to

(a) The xy-plane: ______ and ______

(b) The yz-plane: ______ and ______

(c) The xz-plane: ______ and ______

SOLUTION

The sphere has center at $(1,1,5)$ and radius 5.

(a) The planes parallel to the xy-plane just touching the sphere are 5 above and 5 below the center. Thus, the planes $z=10$ and $z=0$ are both parallel to the xy-plane and touch the sphere at the points $(1,1,10)$ and $(1,1,0)$.

(b) The planes parallel to the yz-plane just touching the sphere are 5 to the left of and 5 to the right of the center. Thus, the planes $x=6$ and $x=-4$ are both parallel to the yz-plane and touch the sphere at the points $(6,1,5)$ and $(-4,1,5)$.

(c) The planes parallel to the xz-plane just touching the sphere are 5 to the left of and 5 to the right of the center. Thus, the planes $y=6$ and $y=-4$ are both parallel to the xz-plane and touch the sphere at the points $(1,6,5)$ and $(1,-4,5)$.

Correct Answers:

- `z = 10`
- `z = 0`
- `x = 6`
- `x = -4`
- `y = 6`
- `y = -4`

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2. **(0 pts) Library/Michigan/Chap12Sec5/Q19.pg**

Find a function $f(x,y,z)$ whose level surface $f=1$ is the graph of the function $g(x,y)=x+6y$.

$f(x,y,z) =$ ______________

SOLUTION

The graph of $g(x,y)=x+6y$ is the set of all points (x,y,z) satisfying $z=x+6y$, or $x+6y-z=0$. This is a level surface, but we want the surface equal to the constant value 1, not 0, so we can add 1 to both sides to get $x+6y-z+1=1$. Thus, $f(x,y,z)=x+6y-z+1$ has level surface $f=1$ identical to the graph of $g(x,y)=x+6y$.

Of course, we could also say that the graph of $g(x,y)$ is equivalent to $z-x+6y=0$, so that $f=z-x-6y+1$.

Correct Answers:

- `x+6*y-z+1`

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3. **(0 pts) Library/CU/Michigan/Chap12Sec6/Q19-hilljb.pg**

For the function $f(x,y)$ below, determine whether there is a value for c making the function continuous everywhere. If so, find it.

$$f(x,y) = \begin{cases} c+y & x \le 4, \\ 6-y & x > 4 \end{cases}$$

$c =$ __________

(If there is no value of c that works, enter **none***, and be sure that you can explain why there is no such value.)*

SOLUTION

The function f is continuous at all points (x,y) with $x \neq 4$. So let's analyze the continuity of f at the point $(4,a)$. We have

$$\lim_{\substack{(x,y)\to(4,a) \\ x<4}} f(x,y) = \lim_{y\to a}(c+y) = c+a$$

and

$$\lim_{\substack{(x,y)\to(4,a) \\ x>4}} f(x,y) = \lim_{y\to a}(6-y) = 6-a.$$

So we need to see if we can find one value for c such that $c+a=5-a$ for all a. This would require that $c=5-2a$, but then c would depend on a, which is exactly what we don't want. Therefore, we cannot make the function continuous everywhere.

Graphically we can see this by thinking about what the function looks like: for $x \le 4$, it is a plane with positive y-slope, passing through the line $y=0$, $z=c$. For $x>4$, it is a plane with negative y-slope, pssing through the line $y=0$, $z=y_0=6$.

Correct Answers:

- `none`

4. **(0 pts) Library/Michigan/Chap12Sec6/Q15.pg**
Show that the function

$$f(x,y) = \frac{x^4 y}{x^8 + y^4}.$$

does not have a limit at $(0,0)$ by examining the following limits.

(a) Find the limit of f as $(x,y) \to (0,0)$ along the line $y = x$.
$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} f(x,y) =$ ________________

(b) Find the limit of f as $(x,y) \to (0,0)$ along the line $y = x^4$.
$\lim_{\substack{(x,y)\to(0,0)\\ y=x^4}} f(x,y) =$ ________________

(Be sure that you are able to explain why the results in **(a)** *and* **(b)** *indicate that f does not have a limit at (0,0)!*

SOLUTION

(a) Let us suppose that (x,y) approaches $(0,0)$ along the line $y = x$. Then

$$f(x,y) = f(x,x) = \frac{x^5}{x^8 + x^4} = \frac{x}{x^4 + 1}.$$

Therefore

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x}} f(x,y) = \lim_{x\to 0} \frac{x}{x^4+1} = 0.$$

(b) On the other hand, if (x,y) approaches 99 990,0) the curve $y = x^4$ we have

$$f(x,y) = f(x,x^4) = \frac{x^8}{x^8 + x^{16}} = 1$$

and so

$$\lim_{\substack{(x,y)\to(0,0)\\ y=x^4}} f(x,y) = \lim_{x\to 0} f(x,x^4) = 1.$$

Thus no matter how close they are to the origin, there will be points (x,y) such that $f(x,y)$ is close to 0 and points (x,y) such that $f(x,y)$ is close to 1. So the limit

$$\lim_{(x,y)\to(0,0)} f(x,y)$$

does not exist.

Correct Answers:

- `0`
- `1`

5. **(0 pts) Library/Michigan/Chap13Sec3/Q17.pg**
(a) Find a vector $\vec{n}$ perpendicular to the plane

$$z = 8x + 6y.$$

$\vec{n} =$ ________________

(b) Find a vector $\vec{v}$ parallel to the plane.
$\vec{v} =$ ________________

SOLUTION

(a) Writing the plane in the form $8x + 6y - z = 0$ shows that a normal vector is

$$\vec{n} = 8\vec{i} + 6\vec{j} - \vec{k}.$$

Any multiple of this vector is also a correct answer.

(b) Any vector perpendicular to $\vec{n}$ is parallel to the plane, so one possible answer is

$$\vec{v} = 6\vec{i} - 8\vec{j}.$$

Many other answers are possible.

Correct Answers:

- `8i+6j-k`
- `6i-8j`

6. **(0 pts) Library/Michigan/Chap13Sec3/Q31.pg**
Compute the angle between the vectors $-\tilde{i} - \tilde{j} + \tilde{k}$ and $-\tilde{i} + \tilde{j} + \tilde{k}$.

angle = ________________ radians

(Give your answer in radians, not degrees.)

SOLUTION

$\cos\theta = \frac{(-\tilde{i}-\tilde{j}+\tilde{k})\cdot(-\tilde{i}+\tilde{j}+\tilde{k})}{||-\tilde{i}-\tilde{j}+\tilde{k}||||-\tilde{i}+\tilde{j}+\tilde{k}||} = \frac{1}{3}$.

So, $\theta = \arccos(\frac{1}{3}) \approx 1.23$ radians.

Correct Answers:

- `arccos(1/3)`

7. **(0 pts) Library/CU/Michigan/Chap13Sec4/Q28.pg**
Suppose $\vec{v} \cdot \vec{w} = 7$ and $||\vec{v} \times \vec{w}|| = 3$, and the angle between $\vec{v}$ and $\vec{w}$ is θ (measured in radians). Find

(a) $\tan\theta =$ ____________________

(b) $\theta =$ ____________________

SOLUTION

(a) Since $\vec{v} \cdot \vec{w} = ||\vec{v}||\,||\vec{w}|| \cos\theta$ and $||\vec{v} \times \vec{w}|| = ||\vec{v}||\,||\vec{w}|| \sin\theta$,

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{||\vec{v} \times \vec{w}||}{\vec{v} \cdot \vec{w}} = \frac{3}{7} = 0.428571.$$

(b) Then $\theta = \tan^{-1}(0.428571) = 0.404892$.

Correct Answers:

- `3/7`
- `arctan(3/7)`

8. **(0 pts) Library/Michigan/Chap13Sec4/Q13.pg**
Find an equation for the plane through the points $(2,4,4), (-2,-1,0), (-2,0,-1)$.

The plane is ________________________

SOLUTION

The displacement vector from $(2,4,4)$ to $(-2,-1,0)$ is:

$$\vec{a} = -4\vec{i} - 5\vec{j} - 4\vec{k}.$$

The displacement vector from $(2,4,4)$ to $(-2,0,-1)$ is:

$$\vec{b} = -4\vec{i} - 4\vec{j} - 5\vec{k}.$$

Therefore a vector normal to the plane is:

$$\vec{n} = \vec{a} \times \vec{b} = 9\vec{i} - 4\vec{j} - 4\vec{k}.$$

Using the first point, the equation of the plane can be written as:

$$9x - 4y - 4z = -14.$$

Correct Answers:

- `9*x-4*y-4*z = -14`

9. (0 pts) Library/Michigan/Chap14Sec3/Q03.pg

Find the equation of the tangent plane to

$$z = e^x + y + y^3 + 9$$

at the point $(0,4,78)$.

$z =$ ________________

SOLUTION

We have

$$z = e^x + y + y^3 + 9.$$

The partial derivatives are

$$\left.\frac{\partial z}{\partial x}\right|_{(x,y)=(0,4)} = e^x \ln(e)\Big|_{(x,y)=(0,4)} = 1,$$

and

$$\left.\frac{\partial z}{\partial y}\right|_{(x,y)=(0,4)} = 1 + 3y^2\Big|_{(x,y)=(0,4)} = 49.$$

So the equation of the tangent plane is

$$z = 78 + 1(x-0) + 49(y-4) = 78 + x + 49(y-4).$$

Correct Answers:

- `78+x+49*(y-4)`

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10. (0 pts) Library/Michigan/Chap14Sec4/Q69.pg

(a) What is the rate of change of $f(x,y) = 3xy + y^2$ at the point $(4,4)$ in the direction $\vec{v} = 2i + 2j$?

$f_{\vec{v}} =$ ________________

(b) What is the direction of maximum rate of change of f at $(4,4)$?

direction = ________________

(Give your answer as a vector.)

(c) What is the maximum rate of change?

maximum rate of change = ________________

SOLUTION

We see that

$$\nabla f = 3yi + (3x+2y)\,j,$$

so at the point $(4,4)$, we have

$$\nabla f = 12i + 20j.$$

(a) The directional derivative is

$$\nabla f(4,4) \cdot \frac{\vec{v}}{|\vec{v}|} = (12i+20j) \cdot \frac{2i+2j}{\sqrt{8}} = \frac{64}{\sqrt{8}}.$$

(b) The direction of maximum rate of change is

$$\nabla f(4,4) = 12i + 20j.$$

(c) The maximum rate of change is

$$|\nabla f(4,4)| = \sqrt{544}.$$

Correct Answers:

- `22.6274`
- `12i+20j`
- `23.3238`

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11. (0 pts) Library/Michigan/Chap14Sec6/Q08.pg

If

$$z = (x+4y)\,e^y, \qquad x = u, \qquad y = \ln(v),$$

find $\partial z/\partial u$ and $\partial z/\partial v$. The variables are restricted to domains on which the functions are defined.

$\partial z/\partial u =$ ________________

$\partial z/\partial v =$ ________________

SOLUTION

Since z is a function of two variables x and y which are functions of two variables the two chain rule identities which apply are:

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

so

$$\frac{\partial z}{\partial u} = (e^y)\,(1) + (4e^y + (x+4y)\,e^y \ln(e)) \cdot 0$$
$$= e^{\ln(v)} \cdot 1.$$

and

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

so

$$\frac{\partial z}{\partial v} = (e^y)(0) + (4e^y + (x+4y)\,e^y \ln(e)) \cdot (\frac{1}{v})$$
$$= \left(4e^{\ln(v)} + (u + 4\ln(v))\,e^{\ln(v)} \ln(e)\right)\frac{1}{v}.$$

Correct Answers:

- `((e^[ln(v)]))*((1))`
- `((4*e^[ln(v)]+[u+4*ln(v)]*e^[ln(v)]*ln(e)))*((1/v))`

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12. (0 pts) Library/Michigan/Chap15Sec1/Q07.pg

Find the critical points for the function

$$f(x,y) = x^2 - 2xy + 3y^2 - 12y$$

and classify each as a local maximum, local minimum, saddle point, or none of these.

critical points: ________________

(give your points as a comma separated list of (x,y) coordinates.)

classifications: ________________

(give your answers in a comma separated list, specifying **maximum**, **minimum**, **saddle point**, *or* **none** *for each, in the same order as you entered your critical points)*

SOLUTION

To find the critical points, we solve $f_x = 0$ and $f_y = 0$ for x and y. These equations are

$$f_x = 2x - 2y = 0,$$

and

$$f_y = -2x + 6y - 12 = 0.$$

We see from the first equation that $x = y$. Substituting this into the second equation shows that $y = 3$. The only critical point is $(3,3)$.

We then have

$$D = (f_{xx})(f_{yy}) - (f_{xy})^2 = (2)(6) - (-2)^2 = 8.$$

Since $D > 0$ and $f_{xx} = 2 > 0$, the function f has a local minimum at the point $(3,3)$.

Correct Answers:

- `(3,3)`
- `minimum`

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13. **(0 pts) Library/Michigan/Chap15Sec2/Q13.pg**
Does the function

$$f(x,y) = \frac{x^2}{2} + 8y^3 + 8y^2 - 5x$$

have a global maximum and global minimum? If it does, identify the value of the maximum and minimum. If it does not, be sure that you are able to explain why.

Global maximum? ____
(Enter the value of the global maximum, or **none** *if there is no global maximum.)*
Global minimum? ____
(Enter the value of the global minimum, or **none** *if there is no global minimum.)*

SOLUTION

Suppose x is fixed. Then for large values of y the sign of f is determined by the highest power of y, namely y^3. Thus, $f(x,y) \to \infty$ as $y \to \infty$, and there can be no global maximum. For $y \to -\infty$ we similarly have $f(x,y) \to -\infty$, so that there is no global minimum either.

Correct Answers:

- `none`
- `none`

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14. **(0 pts) Library/CU/Michigan/Chap15Sec3/Q43.pg**
For each value of λ the function $h(x,y) = x^2 + y^2 - \lambda(2x + 8y - 15)$ has a minimum value $m(\lambda)$.

(a) Find $m(\lambda)$
$m(\lambda) =$ ____________
(Use the letter **L** *for λ in your expression.)*

(b) For which value of λ is $m(\lambda)$ the largest, and what is that maximum value?
$\lambda =$ ________
maximum $m(\lambda) =$ ________

(c) Find the minimum value of $f(x,y) = x^2 + y^2$ subject to the constraint $2x + 8y = 15$ using the method of Lagrange multipliers and evaluate λ.
minimum f = ________
$\lambda =$ ________

(How are these results related to your result in part (b)?)

SOLUTION

(a) The critical points of $h(x,y)$ occur where

$$h_x(x,y) = 2x - 2\lambda = 0,$$

$$h_y(x,y) = 2y - 8\lambda = 0.$$

The only critical point is $(x,y) = (\lambda, 4\lambda)$ and it gives a minimum value for $h(x,y)$. That minimum value is

$$m(\lambda) = h(\lambda, 4\lambda) = \lambda^2 + 16\lambda^2 - \lambda(2\lambda + 32\lambda - 15)$$

or

$$m(\lambda) = -17\lambda^2 + 15\lambda.$$

(b) The maximum value of $m(\lambda) = -17\lambda^2 + 15\lambda$ occurs at a critical point, where $m'(\lambda) = -34\lambda + 15 = 0$. At this point, $\lambda = 0.441176$ and $m(0.441176) = 3.30882$.

(c) We want to minimize $f(x,y) = x^2 + y^2$ subject to the constraint $g(x,y) = 15$, where $g(x,y) = 2x + 8y$. The method of Lagrange multipliers has us solve

$$2x = 2\lambda,$$

$$2y = 8\lambda,$$

and

$$2x + 8y = 15.$$

We can see that this gives the same solutions for x and y as we obtained in part (a), and then, plugging in to the third equation, we have

$$2\lambda + 32\lambda = 15.$$

Thus we get the same λ as before: $\lambda = 0.441176$, so that the minimum value of f is $f(1\lambda, 4\lambda)$, where $\lambda = 0.441176$, or $f_{\text{min}} = 3.30882$.

Note that the two question have the same answer. This makes sense, because in the first problem the largest minimum value will clearly occur when $2x + 8y = 15$, which is the constraint in the third equation.

Correct Answers:

- `15*L - (2*2 + 8*8)*L^2/4`
- `2*15/(2*2 + 8*8)`
- `15*15/(2*2 + 8*8)`
- `15*15/(2*2 + 8*8)`
- `2*15/(2*2 + 8*8)`

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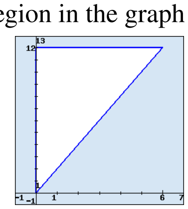
15. **(0 pts) Library/Michigan/Chap16Sec2/Q13.pg**
Consider the shaded region in the graph below.



Write $\int_R f\,dA$ on this region as an iterated integral:
$\int_R f\,dA = \int_a^b \int_c^d f(x,y)\,d\_\_\_\ d\_\_\_$,
where
$a =$ ______,
$b =$ ______,
$c =$ ______, and
$d =$ ______.

SOLUTION

This region lies between $x = 0$ and $x = 6$ and between the lines $y = 2x$ and $y = 12$, and so the iterated integral is

$$\int_0^6 \int_{2x}^{12} f(x,y)\,dy\,dx.$$

Alternatively, we could have set up the integral as follows:

$$\int_0^{12} \int_0^{\frac{y}{2}} f(x,y)\,dx\,dy.$$

Correct Answers:

- `y`
- `x`
- `0`
- `6`
- `2*x`
- `12`

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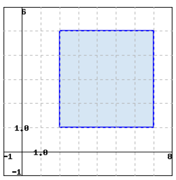
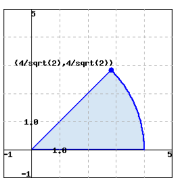
16. **(0 pts) Library/Michigan/Chap16Sec4/Q07.pg**
For each of the following, set up the integral of an arbitrary function $f(x,y)$ over the region in whichever of rectangular or polar coordinates is most appropriate. (Use t for θ in your expressions.)

(a) The region



With $a =$ ___, $b =$ ___,
$c =$ ___, and $d =$ ___,
integral = $\int_a^b \int_c^d$ ________ d___ d___

(b) The region



With $a =$ ___, $b =$ ___,
$c =$ ___, and $d =$ ___,
integral = $\int_a^b \int_c^d$ ________ d___ d___

SOLUTION

(a) Since this is a rectangular region, we use Cartesian coordinates. This gives

$$\int_2^7 \int_1^5 f(x,y)\,dy\,dx.$$

(b) Since this is a partially-circular region, we use polar coordinates. This gives

$$\int_0^{\pi/4} \int_0^4 f(r\cos(\theta), r\sin(\theta))\,r\,dr\,d\theta.$$

Correct Answers:

- `2`
- `7`
- `1`
- `5`
- `f(x,y)`
- `y`
- `x`
- `0`
- `0.785398`
- `0`
- `4`
- `r*f(r*cos(t),r*sin(t))`
- `r`
- `t`

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17. **(0 pts) Library/Michigan/Chap16Sec5/Q09.pg**
Evaluate the triple integral of $f(x,y,z) = \cos(x^2+y^2)$ over the the solid cylinder with height 6 and with base of radius 2 centered on the z axis at $z = -3$.

Integral = ____________________

SOLUTION

We have

$$\int_W f\,dV = \int_{-3}^{-3} \int_0^{2\pi} \int_0^2 (\cos(r^2)\,r\,dr\,d\theta\,dz.$$

Integrating,

$$\int_{-3}^{-3} \int_0^{2\pi} \int_0^2 (\cos(r^2)\,r\,dr\,d\theta\,dz = \int_{-3}^{-3} \int_0^{2\pi} \frac{1}{2} \cdot \sin(r^2)\Big|_{r=0}^{r=2} d\theta\,dz = \int_{-3}^{-3} \int_0^{2\pi}$$

$$\frac{1}{2}(\sin(4)) \int_{-3}^{-3} 2\pi\,dz = 6\pi(\sin(4)).$$

Correct Answers:

- `6*pi*sin(2*2)`

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18. **(0 pts) Library/CU/Michigan/Chap16Sec7/Q19.pg**
In this problem we use the change of variables $x = 3s+t$, $y = s-2t$ to compute the integral $\int_R (x+y)\,dA$, where R is the parallelogram formed by $(0,0)$, $(6,2)$, $(8,-2)$, and $(2,-4)$.

First find the magnitude of the Jacobian, $\left|\frac{\partial(x,y)}{\partial(s,t)}\right| =$ ________.

Then, with $a =$ ________, $b =$ ________,
$c =$ ________, and $d =$ ________,

$\int_R (x+y)\,dA = \int_a^b \int_c^d ($ ___ $s+$ ___ $t+$ ___ $)dt\,ds =$ ____________

SOLUTION

Given

$$\begin{aligned} x &= 3s+t \\ y &= s-2t, \end{aligned}$$

we have

$$\left|\frac{\partial(x,y)}{\partial(s,t)}\right| = \left|\begin{array}{cc} \frac{\partial x}{\partial s} & \frac{\partial x}{\partial t} \\ \frac{\partial y}{\partial s} & \frac{\partial y}{\partial t} \end{array}\right| = \left|\begin{array}{cc} 3 & 1 \\ 1 & -2 \end{array}\right| = |-7|,$$

hence

$$\left|\frac{\partial(x,y)}{\partial(s,t)}\right| = 7.$$

We therefore get

$$\int_R (x+y)\,dA = \int_T ((3s+t)+(s-2t))\left|\frac{\partial(x,y)}{\partial(s,t)}\right| ds dt =$$

$$\int_T (4s-t)(7)\,ds\,dt = \int_T (7(4s-t))\,ds\,dt,$$

where T is the region in the st-plane corresponding to R.

Now, we need to find T.

Because

$$\begin{aligned} x &= 3s+t \\ y &= s-2t \end{aligned}$$

we can use the boundaries of the region R to find the boundaries of T. Using the four vertices of the parallelogram R, we the edges $y=\frac{1}{3}x$, $y=-2x$, $y=\frac{1}{3}(x-2)-4$ and $y=-2(x-6)+2$. So, from the above transformation, the boundaries are $t=0$, $s=0$, $t=2$ and $s=2$. Therefore

$$\int_R (x+y)\,dA=\int_0^2\int_0^2 7(4s-t)\,ds\,dt=84.$$

(Alternately, we could of course also write the integral as $\int_2^0\int_2^0 7(4s-t)\,ds\,dt$.)

Correct Answers:

- `3*2 + 1`
- `0`
- `(2*6 + 2)/(1 + 3*2)`
- `0`
- `(2 - 3*-4)/(3*2 + 1)`
- `7*(3 + 1)`
- `7*(1 - 2)`
- `0`
- `(2*6 + 2)*(2 - 3*-4)*(2 + 2 + 3*2 - 3*-4 + 2*(6 + 3*6 - 2 + 3*-4))/(2*(1 + 3*2)^2)`

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19. **(0 pts) Library/Michigan/Chap16Sec7/Q13.pg**

Find a number a so that the change of variables $s=x+ay, t=y$ transforms the integral $\int\int_R dx\,dy$ over the parallelogram R in the xy-plane with vertices $(0,0)$, $(10,0)$, $(-14,11)$, $(-4,11)$ into an integral

$$\int\int_T \left|\frac{\partial(x,y)}{\partial(s,t)}\right| ds\,dt$$

over a rectangle T in the st-plane.

$a=$ ________

What is $\left|\frac{\partial(x,y)}{\partial(s,t)}\right|$ in this case?

$\left|\frac{\partial(x,y)}{\partial(s,t)}\right|=$ ________

SOLUTION

Inverting the change of variables gives $x=s-at$, $y=t$. The four edges of R are

$$y=0, y=11, y=-\frac{11}{14}x, y=-\frac{11}{14}(x-10).$$

The change of variables transforms the edges to

$$t=0, t=11, t=-\frac{11}{14}s+\frac{11}{14}at, t=-\frac{11}{14}s+\frac{11}{14}at+\frac{55}{7}.$$

These are equations for the edges of a rectangle in the st-plane if the last two are of the form: $s=$ (Constant). This happens when the t terms drop out, or $a=\frac{14}{11}$. With $a=\frac{14}{11}$ the change of variables gives

$$\int\int_T \left|\frac{\partial(x,y)}{\partial(s,t)}\right| ds\,dt$$

over the rectangle

$$T: 0\le t\le 11, 0\le s\le 10.$$

The jacobian $\left|\frac{\partial(x,y)}{\partial(s,t)}\right|$ is

$$\left|\frac{\partial(x,y)}{\partial(s,t)}\right|=\begin{vmatrix}1 & -\frac{14}{11}\\ 0 & 1\end{vmatrix}=1.$$

Correct Answers:

- `-1*-14/11`
- `1`

---

20. **(0 pts) Library/Michigan/Chap17Sec1/Q47.pg**

(a) Find a vector parallel to the line of intersection of the planes $2x+3y-z=0$ and $-2x-5y-4z=7$.

$\vec{v}=$ ________

(b) Show that the point $(1,-1,-1)$ lies on both planes. Then find a vector parametric equation for the line of intersection.

$\vec{r}(t)=$ ________

SOLUTION

(a) Normal vectors to the two planes are

$$\vec{n}_1=2\vec{i}+3\vec{j}-\vec{k} \quad\text{and}\quad \vec{n}_2=-2\vec{i}-5\vec{j}-4\vec{k}.$$

The vector $\vec{n}_1\times\vec{n}_2$ is perpendicular to both planes and parallel to the line of intersection:

$$\vec{n}_1\times\vec{n}_2=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 3 & -1\\ -2 & -5 & -4\end{vmatrix}=-17\tilde{i}+10\tilde{j}-4\tilde{k}.$$

(b) To check that the point $(1,-1,-1)$ lies on the planes, substitute into each equation.

$$2x+3y-z=(2)(1)+(3)(-1)+(-1)(-1)=0,$$

and

$$-2x-5y-4z=(-2)(1)+(-5)(-1)+(-4)(-1)=7.$$

Thus, the point lies on both planes. Then a vector parametric equation of the line is

$$\vec{r}(t)=(1-17t)\,i+(10t-1)\,j-(1+4t)\,k.$$

Correct Answers:

- `-17i+10j-4k`
- `(1,-1,-1)+(-17*i+10*j-4*k)*t`

---

21. **(0 pts) Library/Michigan/Chap17Sec5/Q27.pg**

Find parametric equations for the sphere centered at the origin and with radius 6. Use the parameters s and t in your answer.

$x(s,t)=$ ________,

$y(s,t)=$ ________, and

$z(s,t)=$ ________, where

____ $\le s\le$ ____ and

____ $\le t\le$ ____.

SOLUTION

We use spherical coordinates with $\phi=s$ and $\theta=t$ as the two parameters. Since the radius is 6, we can take

$$x=6\cos(t)\sin(s),\quad y=6\sin(t)\sin(s),\quad\text{and}\quad z=6\cos(s),$$

with

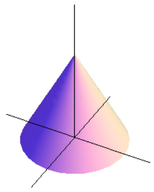
$$0\le s\le\pi \quad\text{and}\quad 0\le t\le 2\pi.$$

Correct Answers:

- `6*cos(t)*sin(s)`
- `6*sin(t)*sin(s)`
- `6*cos(s)`
- `0`
- `2*pi`
- `0`

- pi

22. **(0 pts) Library/Michigan/Chap17Sec5/Q31.pg**
Consider the cone shown below.



If the height of the cone is 9 and the base radius is 4, write a parameterization of the cone in terms of $r = s$ and $\theta = t$.
$x(s,t) =$ ________,
$y(s,t) =$ ________, and
$z(s,t) =$ ________, with
____ $\leq s \leq$ ____ and
____ $\leq t \leq$ ____.
SOLUTION
Since the parameterization is specified to be in terms of the radius r and angle θ, we find x, y and z in terms of the parameters $r = s$ and $\theta = t$. We have

$$x = s\cos t,$$

$$y = s\sin t,$$

and

$$z = 9(1 - \frac{1}{4}s),$$

with

$$0 \leq s \leq 4 \quad \text{and} \quad 0 \leq t \leq 2\pi.$$

There are, of course, other parameterizations, but they will not be in terms of r and θ, as required in this problem.
Correct Answers:

- s*cos(t)
- s*sin(t)
- 9 - (9/4)*s
- 0
- 4
- 0
- 2*pi

23. **(0 pts) Library/Michigan/Chap18Sec1/Q09.pg**
Calculate the line integral of the vector field $\vec{F} = 4\vec{i} - 3\vec{j}$ along the line from the point $(1,0)$ to the point $(7,0)$.
The line integral = ________
SOLUTION
Since $\vec{F}$ is a constant vector field and the curve is a line, $\int_C \vec{F} \cdot d\vec{r} = \vec{F} \cdot \Delta\vec{r}$, where $\Delta\vec{r} = 6\vec{i}$. Therefore,

$$\int_C \vec{F} \cdot d\vec{r} = (4\vec{i} - 3\vec{j}) \cdot 6\vec{i} = 24.$$

Correct Answers:

- (7 - 1)*4

24. **(0 pts) Library/Michigan/Chap18Sec2/Q13.pg**
Find $\int_C \vec{F} \cdot d\vec{r}$ for $\vec{F} = 5y\vec{i} - (\sin y)\vec{j}$ on the curve counterclockwise around the unit circle C starting at the point $(1,0)$.
$\int_C \vec{F} \cdot d\vec{r} =$ ________
SOLUTION
The curve C is parameterized by

$$\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j}, \qquad \text{for } 0 \leq t \leq 2\pi,$$

so,

$$\vec{r}'(t) = -\sin t\,\vec{i} + \cos t\,\vec{j}.$$

Thus,

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} (5\sin t\,\vec{i} - \sin(\sin t)\,\vec{j}) \cdot (-\sin t\,\vec{i} + \cos t\,\vec{j})dt$$

$$= \int_0^{2\pi} (-2\sin^2 t - \sin(\sin t)\cos t)dt$$

$$= \frac{5}{2}(\sin t\cos t - t) + \cos(\sin t)\Big|_0^{2\pi}$$

$$= -5\pi$$

Correct Answers:

- -1*5*pi

25. **(0 pts) Library/Michigan/Chap18Sec3/Q31.pg**
For the vector field $\vec{G} = (ye^{xy} + 3\cos(3x+y))\vec{i} + (xe^{xy} + \cos(3x+y))\vec{j}$, find the line integral of $\vec{G}$ along the curve C from the origin along the x-axis to the point $(5,0)$ and then counterclockwise around the circumference of the circle $x^2 + y^2 = 25$ to the point $(5/\sqrt{2}, 5/\sqrt{2})$.
$\int_C \vec{G} \cdot d\vec{r} =$ ________
SOLUTION
Since $\vec{G} = \nabla(e^{xy} + \sin(3x+y))$, the line integral can be calculated using the Fundamental Theorem of Line Integrals:

$$\int_c \vec{F} \cdot d\vec{r} = e^{xy} + \sin(3x+y)\Big|_{(0.0)}^{(5/\sqrt{2},5/\sqrt{2})} = e^{25/2} + \sin(6\sqrt{2}) - 3.$$

Correct Answers:

- 5/sqrt(2)

26. **(0 pts) Library/CU/Michigan/Chap18Sec4/Q18.pg**

Calculate $\int_C ((8x+7y)\vec{i} + (4x+4y)\vec{j}) \cdot d\vec{r}$ where C is the circular path with center (a,b) and radius m, oriented counterclockwise. Use Green's Theorem.
$\int_C ((8x+7y)\vec{i} + (4x+4y)\vec{j}) \cdot d\vec{r} =$ ________
SOLUTION
Green's theorem gives

$$\int_C (((8x+7y)\vec{i} + (4x+4y)\vec{j}) \cdot d\vec{r} = \iint_R \frac{\partial}{\partial x}(4x+4y) - \frac{\partial}{\partial y}(8x+7y)dA$$

$$= \iint_R -3dA = -3 \cdot \text{Area of } R = -3\pi m^2.$$

Correct Answers:

- (4-7)*pi*m^2

27. **(0 pts) Library/Michigan/Chap18Sec4/Q15.pg**
Use Green's Theorem to calculate the circulation of $\vec{F} = 2xy\vec{i}$ around the rectangle $0 \le x \le 3$, $0 \le y \le 8$, oriented counterclockwise.

circulation = ________________

SOLUTION

By Green's Theorem, with R representing the interior of the square,

$$\int_C \vec{F} \cdot d\vec{r} = \int_R \left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial y}(2xy) \right) dA = \int_R -2x\,dA.$$

Thus the circulation is

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^3 \int_0^8 -2x\,dy\,dx = -72.$$

Correct Answers:

- `-2*3^2*8/2`

---

28. **(0 pts) Library/Michigan/Chap19Sec1/Q07.pg**
Find the flux of the constant vector field $\vec{v} = -2\vec{i} + 4\vec{j} - 2\vec{k}$ through a square plate of area 9 in the zx-plane oriented in the positive y-direction.

flux = ________________

SOLUTION

On the surface, $d\vec{A} = \vec{j}\,dA$, so only the $\vec{j}$ component of $\vec{v}$ contributes to the flux:

$$\text{Flux } = \int_S \vec{v} \cdot d\vec{A} = \int_S (-2\vec{i} + 4\vec{j} - 2\vec{k}) \cdot \vec{j}\,dA = 4 \cdot 9 = 36.$$

Correct Answers:

- `9*4`

---

29. **(0 pts) Library/Michigan/Chap19Sec1/Q21.pg**
Calculate the flux of the vector field $\vec{F} = 6\vec{i} + x^2\vec{j} - 4\vec{k}$, through the square of side 4 in the plane $y = 5$, centered on the y-axis, with sides parallel to the x and z axes, and oriented in the positive y-direction.

flux = ________________

SOLUTION

The only contribution to the flux is from the $\vec{j}$-component, and since $d\vec{A} = \vec{j}\,dx\,dz$ on the square, S, we have

$$\text{Flux } = \int_S (6\vec{i} + x^2\vec{j} - 4\vec{k}) \cdot d\vec{A} = \int_{-2}^{2}\int_{-2}^{2} x^2\vec{j}\cdot\vec{j}\,dx\,dz = \int_{-2}^{2} \frac{x^3}{3}\bigg|_{-2}^{2} dz = \frac{16}{3}\cdot 4 = \frac{64}{3}.$$

Correct Answers:

- `1*4*2*(2)^3/3`

---

30. **(0 pts) Library/Michigan/Chap19Sec2/Q03.pg**
Calculate the flux integral. $\int_S (3\vec{i} + 5z\vec{k}) \cdot d\vec{A}$ where S is $z = x^2 + y^2$ with $0 \le x \le 3, 0 \le y \le 3$, oriented upward.

$\int_S (3\vec{i} + 5z\vec{k}) \cdot d\vec{A} =$ ________________

SOLUTION

Since $z = x^2 + y^2$, we have $z_x = 2x$ and $z_y = 2y$, so $d\vec{A} = (-2x\vec{i} - 2y\vec{j} + \vec{k})\,dx\,dy$. Thus

$$\int_S (3\vec{i} + 5z\vec{k}) \cdot d\vec{A} = \int_0^3 \int_0^3 (3\vec{i} + 5(x^2+y^2)\vec{k}) \cdot (-2x\vec{i} - 2y\vec{j} + \vec{k})\,dx\,dy,$$

$$= \int_0^3 \int_0^3 (-6x + 5x^2 + 5y^2)\,dx\,dy = \int_0^3 \left(-3x^2 + \frac{5}{3}x^3 + 5xy^2 \right)\bigg|_0^3 dy$$

$$= \int_0^3 18 + 15y^2\,dy = 18y + \frac{0}{3}y^3\bigg|_0^3 = 189.$$

Correct Answers:

- `-3*3^2*3 + 5*3^3*3/3 + 5*3*3^3/3`

---

31. **(0 pts) Library/Michigan/Chap19Sec2/Q11.pg**
Compute the flux of the vector field $\vec{F} = 5x\vec{i} + 5y\vec{j}$ through the surface S, which is the part of the surface $z = 36 - (x^2 + y^2)$ above the disk of radius 6 centered at the origin, oriented upward.

flux = ________________

SOLUTION

Writing the surface S as $z = f(x,y) = 36 - x^2 - y^2$, we have

$$d\vec{A} = (-f_x\vec{i} - f_y\vec{j} + \vec{k})dx\,dy. = (2x\vec{i} + 2y\vec{j} + \vec{k})dx\,dy.$$

Thus,

$$\int_S \vec{F} \cdot d\vec{A} = \int_R 5(x\vec{i} + y\vec{j}) \cdot (2x\vec{i} + 2y\vec{j} + \vec{k})\,dx\,dy = \int_R 10(x^2+y^2)\,dx\,dy.$$

Converting to polar coordinates, we have

$$\text{flux} = \int_0^{2\pi}\int_0^6 10r^2\,r\,dr\,d\theta = (2\pi)(\frac{5}{2})r^4\bigg|_0^6 = 6480\pi.$$

Correct Answers:

- `pi*5*1296`

---

32. **(0 pts) Library/CU/Michigan/Chap19Sec3/Q09b.pg**
Find the surface area of the region S on the plane $z = 8x + 6y$ such that $0 \le x \le 15$ and $0 \le y \le 10$ by finding a parameterization of the surface and then calculating the surface area.

A parameterization is

$x(s,t) =$ ________________,
$y(s,t) =$ ________________, and
$z(s,t) =$ ________________, with
___ $\le s \le$ ___ and
___ $\le t \le$ ___.

Then, the surface area = ________________

SOLUTION

A parameterization of S is

$$x = s, \quad y = t, \quad z = 8s + 6t, \quad \text{for } 0 \le s \le 15, \quad 0 \le t \le 10.$$

We compute

$$\frac{\partial\vec{r}}{\partial s} \times \frac{\partial\vec{r}}{\partial t} = (\vec{i} + 8\vec{k}) \times (\vec{j} + 6\vec{k}) = -8\vec{i} - 6\vec{j} + \vec{k},$$

so that

$$\left\| \frac{\partial\vec{r}}{\partial s} \times \frac{\partial\vec{r}}{\partial t} \right\| = \sqrt{101}.$$

Then

$$\text{Surface area} = \int_S dA = \int_R \left\| \frac{\partial\vec{r}}{\partial s} \times \frac{\partial\vec{r}}{\partial t} \right\| dA = \int_{t=0}^{10}\int_{s=0}^{15} \sqrt{101}\,ds\,dt = 150\sqrt{10$$

Correct Answers:

- `s`
- `t`

- `8*s + 6*t`
- `0`
- `15`
- `0`
- `10`
- `sqrt(8*8 + 6*6 + 1)*15*10`

---

33. **(0 pts) Library/Michigan/Chap19Sec3/Q01.pg**

Compute the flux of the vector field $\vec{F} = z\vec{k}$ through the parameterized surface S, which is oriented toward the z-axis and given, for $0 \le s \le 3$, $0 \le t \le 4$, by

$$x = 5s + 5t, \quad y = 5s - 5t, \quad z = s^2 + t^2.$$

flux = ________

SOLUTION

Since S is given by

$$\vec{r}(s,t) = (5s+5t)\vec{i} + (5s-5t)\vec{j} + (s^2+t^2)\vec{k},$$

we have

$$\frac{\partial \vec{r}}{\partial s} = 5\vec{i} + 5\vec{j} + 2s\vec{k} \quad \text{and} \quad \frac{\partial \vec{r}}{\partial t} = 5\vec{i} - 5\vec{j} + 2t\vec{k},$$

and

$$\frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 5 & 5 & 2s \\ 5 & -5 & 2t \end{vmatrix} = (10s+10t)\vec{i} + (10s-10t)\vec{j} - 50\vec{k}.$$

Since the $\vec{i}$ component of this vector is positive for $0 < s < 3$, $0 < t < 4$, it points away from the z-axis, and so has the opposite orientation to the one specified. Thus, we use

$$d\vec{A} = -\frac{\partial \vec{r}}{\partial s} \times \frac{\partial \vec{r}}{\partial t}\, ds\, dt,$$

and so we have

$$\int_S \vec{F} \cdot d\vec{A} = -\int_0^4 \int_0^3 (s^2+t^2)\vec{k} \cdot \left((10s+10t)\vec{i} + (10s-10t)\vec{j} - 50\vec{k}\right) ds\, dt$$

$$= 50 \int_0^4 \int_0^3 (s^2+t^2)\, ds\, dt = 50 \int_0^4 \left(\frac{s^3}{3} + st^2\right)\Bigg|_{s=0}^{s=3} dt$$

$$= 50 \int_0^4 (9+3t^2)\, dt = 50(9t+t^3\Big|_0^4 = 50(36+64) = 5000.$$

Correct Answers:

- `2*5*5*3*4*(3*3 + 4*4)/3`

---

34. **(0 pts) Library/CU/Michigan/Chap20Sec1/Q17.pg**

A smooth vector field $\vec{F}$ has div $\vec{F}(3,2,1) = 3$. Estimate the flux of $\vec{F}$ out of a small sphere of radius 0.005 centered at the point $(3,2,1)$.

flux ≈ ________

SOLUTION

Since div $F(3,2,1)$ is the flux density out of a small region surrounding the point $(3,2,1)$, we have

$$\text{div}\, \vec{F}(3,2,1) \approx \frac{\text{Flux out of small region around } (3,2,1)}{\text{Volume of region.}}$$

So

$$\text{Flux out of region } \approx (\text{div}\, \vec{F}(3,2,1)) \cdot \text{Volume of region}$$

$$= 3 \cdot \frac{4}{3}\pi(0.005)^3 = 1.5708 \times 10^{-6}.$$

Correct Answers:

- `3*4*pi*(0.005)^3/3`

---

35. **(0 pts) Library/Michigan/Chap20Sec1/Q01.pg**

Consider div $\left(\dfrac{-y\vec{i} + x\vec{j}}{(x^2+y^2)}\right)$.

(a) Is this a vector or a scalar? [?]

(b) Calculate it:

div $\left(\frac{-y\vec{i}+x\vec{j}}{(x^2+y^2)}\right)$ = ________ + ________ = ________

SOLUTION

(a) div $\left(\frac{-y\vec{i}+x\vec{j}}{(x^2+y^2)}\right)$ is a scalar

(b) We have

$$\text{div}\left(\frac{-y\vec{i}+x\vec{j}}{(x^2+y^2)}\right) = \frac{2xy}{(x^2+y^2)^2} + \frac{(-2)xy}{(x^2+y^2)^2} = 0.$$

Correct Answers:

- `scalar`
- `2*1*x*y/(x^2 + y^2)^(1+1)`
- `-2*1*x*y/(x^2 + y^2)^(1+1)`
- `0`

---

36. **(0 pts) Library/Michigan/Chap20Sec2/Q07.pg**

Use the Divergence Theorem to calculate the flux of the vector fields $\vec{F}_1 = -3z\vec{i} + 3x\vec{k}$ and $\vec{F}_2 = -3z\vec{i} + (8-3y)\vec{j} + 3x\vec{k}$ through the surface S given by the sphere of radius a centered at the origin with outwards orientation. Be sure that you are able to explain your answers geometrically.

With W giving the interior of the sphere,

$\int_S \vec{F}_1 \cdot d\vec{A} = \int_W$ ________ $dV =$ ________

and

$\int_S \vec{F}_2 \cdot d\vec{A} = \int_W$ ________ $dV =$ ________

SOLUTION

The divergence of the fields are

$$\text{div}\, \vec{F}_1 = \frac{\partial}{\partial x}(-3z) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(3x) = 0.$$

and

$$\text{div}\, \vec{F}_2 = \frac{\partial}{\partial x}(-3z) + \frac{\partial}{\partial y}(8-3y) + \frac{\partial}{\partial z}(3x) = -3.$$

Hence,

$$\int_S \vec{F}_1 \cdot d\vec{A} = \int_W 0\, dV = 0,$$

and

$$\int_S \vec{F}_2 \cdot d\vec{A} = \int_W -3\, dV = -3 \int_W dV = -4\pi a^3.$$

This makes sense, because, the vector field $\vec{F}_1$ is flowing around the y-axis and is therefore always tangent to the sphere, so that the flux is always zero. $\vec{F}_2$, however, adds a component in the y direction that points in for $y > 0$ and out (for sufficiently large negative y) for $y < 0$, and therefore results in a non-zero flux.

Correct Answers:

- `0`

- 0
- -3
- 4*-3*pi*a^3/3

---

37. **(0 pts) Library/Michigan/Chap20Sec3/Q02.pg**
Compute the curl of the vector field $\vec{F} = -3zi + 7yj - 4xk$.

curl = ________________

SOLUTION

We have

$$\text{curl}(-3zi + 7yj - 4xk) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -3z & 7y & -4x \end{vmatrix} = \vec{j}.$$

Correct Answers:

- j

---

38. **(0 pts) Library/Michigan/Chap20Sec3/Q19.pg**
Three small squares, S_1, S_2, and S_3, each with side 0.05 and centered at the point $(4,8,1)$, lie parallel to the xy-, yz- and xz-planes, respectively. The squares are oriented counterclockwise when viewed from the positive z-, x-, and y-axes, respectively. A vector field $\vec{G}$ has circulation around S_1 of -0.0075, around S_2 of 0.25, and around S_3 of 1.25. Estimate curl $\vec{G}$ at the point $(4,8,1)$

curl$\vec{G} \approx$ ________________

SOLUTION

The vector curl $\vec{G}$ has its component in the x-direction given by

$$(\text{curl } \vec{G})_x \approx \frac{\text{Circulation around small square around } x\text{-axis}}{\text{Area inside square}}$$

$$= \frac{\text{Circulation around } S_2}{\text{Area inside } S_2} = \frac{0.25}{(0.05)^2} = 100.$$

Similar reasoning leads to

$$(\text{curl } \vec{G})_y \approx \frac{\text{Circulation around } S_3}{\text{Area inside } S_3} = \frac{1.25}{(0.05)^2} = 500.$$

and

$$(\text{curl } \vec{G})_z \approx \frac{\text{Circulation around } S_1}{\text{Area inside } S_1} = \frac{-0.0075}{(0.05)^2} = -3.$$

Thus,

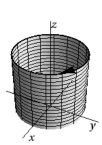
$$\text{curl } \vec{G} \approx 100\vec{i} + 500\vec{j} - 3\vec{k}.$$

Correct Answers:

- 100i+500j-3k

---

39. **(0 pts) Library/CU/Michigan/Chap20Sec4/Q17.pg**
The figure below open cylindrical can, S, standing on the xy-plane. (S has a bottom and sides, but no top.)



The side of S is given by $x^2 + y^2 = 9$, and its height is 3.

(a) Give a parametric equation, $\vec{r}(t)$ for the rim, C.

$\vec{r}(t) =$ ________________ **(You must use angle bracket notation and enter a 3-vector)**, with ___ $\leq t \leq$ ___.

(b) If S is oriented outward and downward, find $\int_S \text{curl } (-5y\vec{i} + 5x\vec{j} + 6z\vec{k}) \cdot d\vec{A}$.

$\int_S \text{curl } (-5y\vec{i} + 5x\vec{j} + 6z\vec{k}) \cdot d\vec{A} =$ ________________

SOLUTION

(a) The equation of the rim, C, is $x^2 + y^2 = 9$, $z = 0$. This is a circle of radius 3 centered on the z-axis, and lying in the plane $z = 2$. We can parameterize this as

$$\vec{r}(t) = 3\cos(t)\,i - 3\sin(t)\,j + 3k, \quad 0 \leq t \leq 2\pi.$$

(b) Use Stokes' Theorem, with C oriented clockwise when viewed from above:

$$\int_S \text{curl } (-5y\vec{i} + 5x\vec{j} + 6z\vec{k}) \cdot d\vec{A} = \int_C (-5y\vec{i} + 5x\vec{j} + 6z\vec{k}) \cdot d\vec{r}.$$

Since C is horizontal, the $\vec{k}$ component does not contribute to the integral. The remaining vector field, $-5y\vec{i} + 5x\vec{j}$, is tangent to C, of constant magnitude $||-5y\vec{i} + 5x\vec{j}|| = 15$ on C, and points in the opposite direction to the orientation. Thus

$$\int_S \text{curl } (-5y\vec{i} + 5x\vec{j} + 6z\vec{k}) \cdot d\vec{A} = \int_C (-5y\vec{i} + 5x\vec{j}) \cdot d\vec{r}$$
$$= -15 \cdot \text{ Length of curve } = -15 \cdot 2\pi 3 = -90\pi.$$

Correct Answers:

- 3*cos(t)i-3*sin(t)j+3k
- 0
- 2*pi
- -2*pi*5*3^2

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40. **(0 pts) Library/Michigan/Chap20Sec4/Q15.pg**
Find $\int_C \vec{F} \cdot d\vec{r}$ where C is a circle of radius 3 in the plane $x + y + z = 4$, centered at $(4,1,-1)$ and oriented clockwise when viewed from the origin, if $\vec{F} = -4z\vec{j} + 6y\vec{k}$

$\int_C \vec{F} \cdot d\vec{r} =$ ________________

SOLUTION

Since

$$\text{curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & -4z & 6y \end{vmatrix} = 10\vec{i},$$

writing S for the disk in the plane enclosed by the circle, Stokes' Theorem gives

$$\int_C \vec{F} \cdot d\vec{r} = \int_S \text{curl } \vec{F} \cdot d\vec{A} = \int_S 10\vec{i} \cdot d\vec{A}.$$

Now $d\vec{A} = \vec{n}\,dA$, where $\vec{n}$ is the unit vector perpendicular to the plane, so

$$\vec{n} = \frac{1}{\sqrt{3}}(\vec{i} + \vec{j} + \vec{k}).$$

Thus

$$\int_C \vec{F}\,d\vec{r} = \int_S 10\vec{i} \cdot \frac{\vec{i} + \vec{j} + \vec{k}}{\sqrt{3}}\,dA = \int_S \frac{10}{\sqrt{3}}\,dA = \frac{10}{\sqrt{3}} \cdot \text{Area of disk } = \frac{10}{\sqrt{3}}\pi 3^2 =$$

Correct Answers:

- 10*pi*3*3/sqrt(3)